5.3 Newton's Second Law of Motion

Newton's Second Law of Motion

Key ideas

- Newton's second law states that the acceleration of a system is equal to the net external force on the system divided by its mass.
- Newton's second law is a vector relationship where the acceleration is in the same direction as the net force.
- Newton's second law can be used for finding acceleration using known forces or for finding unknown forces, given an acceleration and some forces acting on the system.

Learning objectives

After completing this section, you should be able to...

- state Newton's second law,
- determine the acceleration of a system, given the forces acting on it,
- determine an unknown force on a system, given other forces on it and its acceleration, and
- define equilibrium and static equilibrium.

Let's return to an example we introduced in Section 5.1 (Forces). A man is pushing a box on a sheet of ice. Let's assume the box is initially at rest. We also assume the ice is slippery enough that we can ignore any frictional force it applies to the box.

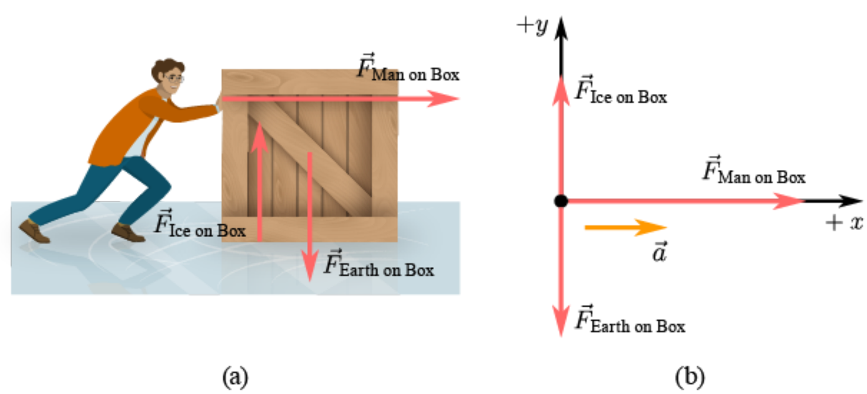

Figure 5.7 (a) A man pushes a box on a sheet of ice. (b) The free-body diagram of the forces acting on the box. The box's acceleration is also indicated.

We have established that the surface of the Earth is a reasonably good inertial reference frame, so Newton's first law applies. As described in Example 5.2, the two vertical forces add, as vectors, to zero, and the box has a net force on it to the right. The reason the two vertical forces must add to zero will be further discussed below. The net force is only due to $\vec{F}_{\text{man on box}}$ in the $+x$-direction. From Newton's first law, if there is a net force on the box in an inertial reference frame, the box must be accelerating. We would expect the acceleration to be in the same direction as the net force, that is, the $+x$-direction.

What would happen if the man pushed with a greater force? We would expect the acceleration to be greater as well. For example, if the man pushed twice as hard, doubling the magnitude of the net force on the box, the box's acceleration would be twice as large.

But what if the man pushed a larger box? As we discussed in Section 5.1 (Newton's First Law of Motion), objects with greater mass have a greater amount of inertia. That is, if we apply the same net force to two different objects, the object with the greater mass will experience a lower acceleration than the one with the smaller mass. In fact, if the man pushed a second box with twice the mass of the first with the same net force, we expect the acceleration of the second box would be half as much.

Our example has demonstrated some important facts about the relationship between an object's acceleration and the net force acting on it:

1. An object's (vector) acceleration is in the same direction as the net force acting on it;
2. The magnitude of the object's acceleration is directly proportional to the magnitude of the net force on it, and;
3. The magnitude of the object's acceleration is inversely proportional to the object's mass.

These facts have been confirmed by countless experiments and applications over the centuries. Furthermore, it turns out that these facts can be generalized to a system of multiple objects, provided we treat the system as a particle, as we described in Section 5.1 (Forces). Note: a full proof of why it works for multi-object systems actually requires Newton's third law, which we'll soon discuss.

Isaac Newton encapsulated these facts into his second law of motion, which in modern language is described using a vector equation.

Newton's Second Law of Motion

The acceleration $\vec{a}$ of a system is directly proportional to and in the same direction as the net external force $\vec{F}_{net}$ on the system and inversely proportional to its mass m. In equation form, Newton's second law is:

$$\vec{a} = \frac{\vec{F}_{net}}{m} \tag{5.2}$$

As a vector equation, Newton's second law is really three scalar equations in one, one for each Cartesian component. That is:

$$a_x = \frac{F_{net,x}}{m},\ a_y = \frac{F_{net,y}}{m},\ a_z = \frac{F_{net,y}}{m} \tag{5.3}$$

Furthermore, we can take the magnitude of both sides of the vector equation, again showing that the magnitude of acceleration is related to the magnitude of the net force:

$$a = \frac{F_{net}}{m} \tag{5.4}$$

Newton's second law of motion is often written as:

$$\vec{F}_{net} = m\vec{a} \tag{5.5}$$

where $\vec{F}_{net} = \sum \vec{F}$, that is, the vector sum of all external forces acting on the system.

Newton's second law now explains the definition of the SI unit of force, the newton, that we gave in Section 5.1 (Forces). We define $1\ \text{N}$ using $F_{net} = ma$, such that:

$$1\ \text{N} = 1\ \text{kg} \cdot 1\ \text{m/s}^2$$

Example 5.5

Acceleration of a Model Rocket

A model rocket has a mass of 0.500 kg, and the gravitational force by the Earth on the rocket is 4.91 N. The rocket's engine applies a force called **thrust** to the rocket with a magnitude of 40.0 N at an angle of 30.0° above the horizontal plane. Ignoring air resistance, find the magnitude and direction of the rocket's acceleration.

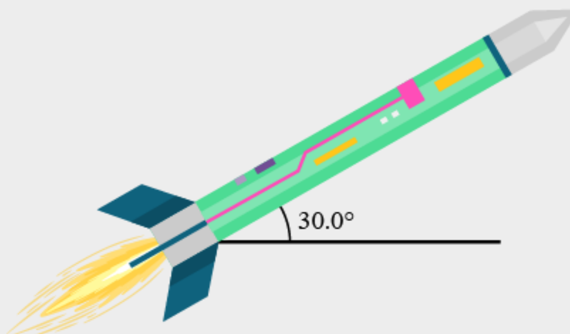

Figure 5.8 A model rocket is shown in flight.

Strategize

Our system is the rocket. We will first draw a free-body diagram of the rocket, taking into account all the forces acting on it. We will need to find the forces in terms of their components and add them as vectors to find the net force. Once we have the components of the net force, we can apply Newton's second law to find the components of the acceleration. We can then use vector algebra to find the acceleration magnitude and direction.

Develop and Solve

The forces on the rocket are the force by the engine on the rocket, $\vec{F}_{\text{eng on r}}$, oriented up and to the right at 30.0° above the horizontal, and the gravitational force on the rocket by the Earth, $\vec{F}_{\text{Earth on r}}$, pointing downward. We will choose the $+x$-direction to be to the right and the $+y$-direction to be upward. Figure 5.9 shows the free-body diagram for the rocket, with figure (a) representing the forces on the rocket and figure (b) redrawing $\vec{F}_{\text{eng on r}}$ in terms of its x- and y-components. Note that $\vec{F}_{\text{Earth on r}}$ only has a nonzero y-component.